

# 6

## Exploring randomness

### Answers

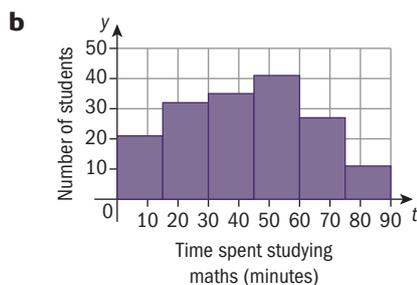
#### Skills check

- 1** Write in ascending order: 85, 88, 91, 94, 95, 96, 97, 103, 103, 107, 110, 114
- a** Median is between 96 and 97 = 96.5 kg
- b** Mode = 103 kg (most frequent)
- c** Mean = total ÷ number of observations  
 $= 1183 \div 12$   
 $= 98.6$  kg
- d** Range = 114 – 85 = 29 kg
- e** Lower quartile =  $\frac{13}{4}$ th observation  
 $= 91 + \frac{1}{4} \times 3 = 91.75$  kg
- Upper quartile =  $\frac{3}{4} \times 13$ th observation  
 $= 9\frac{3}{4}$ th observation  
 $= 103 + \frac{3}{4} \times 4$   
 $= 106$  kg
- f** IQR = 106 – 91.75 = 14.25 kg

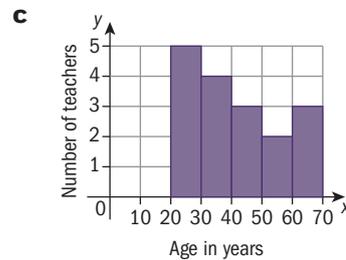
- 2 a**  $\binom{8}{3} = \frac{8 \times 7 \times 6}{3!} = 8 \times 7 = 56$
- b** Total number of ways – number of ways where all 3 have brown eyes  
 $= \binom{20}{3} - \binom{12}{3} = 920$

#### Exercise 6A

- 1 a** Discrete (as they are asked for an answer in whole minutes)



- 2 a** Continuous
- b**  $5 + 4 + 3 + 2 + 3 = 17$



- 3 a** Continuous

**b**

Mass (kg)	Number of chickens
$1 \leq w < 2$	8
$2 \leq w < 3$	24
$3 \leq w < 4$	50
$4 \leq w < 5$	14

- c**  $8 + 24 + 50 + 14 = 96$

- 4 a** Continuous

**b**

Time to get home (mins)	Number of students
$5 \leq t < 10$	1
$10 \leq t < 15$	2
$15 \leq t < 20$	4
$20 \leq t < 25$	4
$25 \leq t < 30$	2
$30 \leq t < 35$	2
$35 \leq t < 40$	1
$40 \leq t < 45$	1

- c** 5 mins

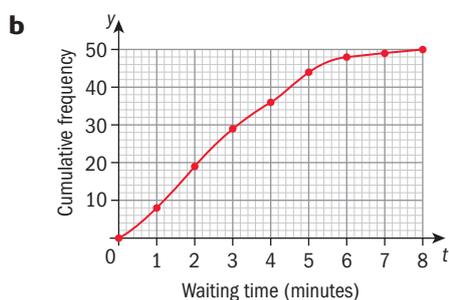
- 5 a** First diagram = D  
 Second diagram = A  
 Third diagram = C

### Exercise 6B

- 1 a 1 goal (highest frequency of 7)  
 b  $170 \leq h < 180$  (highest frequency of 10)

2 a

t(minutes)	Frequency	CF
$0 \leq t < 1$	8	8
$1 \leq t < 2$	11	19
$2 \leq t < 3$	10	29
$3 \leq t < 4$	7	36
$4 \leq t < 5$	8	44
$5 \leq t < 6$	4	48
$6 \leq t < 7$	1	49
$7 \leq t < 8$	1	50
	50	



% waiting longer than 5 minutes  
 $= \frac{6}{50} \times 100\% = 12\%$

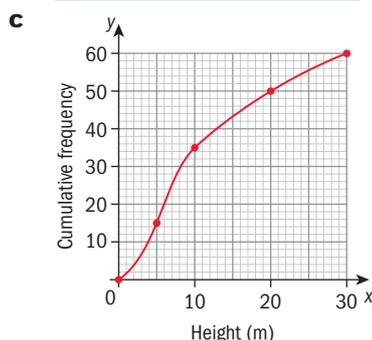
- c Estimates from table and graph:  
 Mean  $\approx 2.8$  mins, Median  $\approx 2.6$  mins  
 Modal interval is  $1 \leq t < 2$  mins

3 a

Height (m)	Frequency
$0 \leq h < 5$	15
$5 \leq h < 10$	20
$10 \leq h < 20$	15
$20 \leq h < 30$	10
	60

b

Height (m)	CF
5	15
10	35
20	50
30	60



- c Number less than 18 m = 47  
 $\therefore \% < 18 \text{ m} = \frac{13}{60} \times 100\% \approx 22\%$   
 d Mean  $\approx 11.0$  m  
 Median  $\approx 9$  m  
 Modal class is  $5 \leq h < 10$  m

- 4 a Mode = 3  
 Median = 3  
 b Use:  $\frac{30+a}{8} = \frac{a+3}{2}$   
 or  $\frac{30+a}{8} = \frac{3+6}{2}$   
 Both give  $a = 6$   
 This makes the set bimodal at 3 and 6.

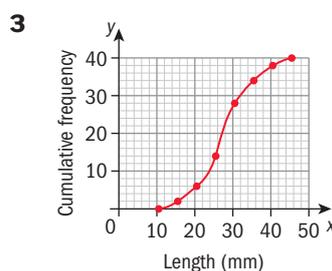
- 5 a Series is  $\ln a + \frac{1}{2} \ln a + \frac{1}{4} \ln a + \frac{1}{8} \ln a + \dots$   
 This is a GP with common ratio  $r = \frac{1}{2}$   
 Since  $|r| < 1$ , this converges with sum  
 $= \frac{A}{1-r} = \frac{\ln a}{1-\frac{1}{2}} = 2 \ln a$

b Mean =  $\frac{\sum_{r=1}^n 2^{\frac{r-1}{2}} \ln a}{n} = \frac{A(1-r^n)}{(1-r)n}$   
 $= \frac{\ln a \left(1 - \frac{1}{2^n}\right)}{\frac{1}{2}n} = \frac{2 \ln a \left(1 - \frac{1}{2^n}\right)}{n}$

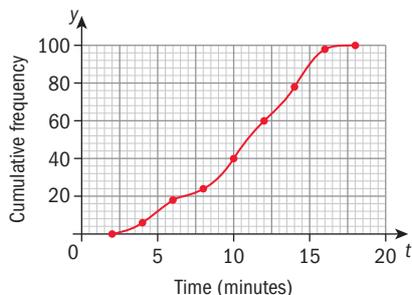
- c Need  $\frac{2 \ln a}{n} \left(1 - \frac{1}{2^n}\right) < 0.01 \ln a$   
 $\Rightarrow \left(1 - \frac{1}{2^n}\right) < 0.005 n$   
 $\Rightarrow n = 200$

### Exercise 6C

- 1 Arrange in order: 30, 45, 55, 60, 65, 65, 70, 75, 75, 110, 120, 125  
 a Range =  $125 - 30 = 95$  cm  
 b Median =  $6\frac{1}{2}$ th reading =  $\frac{65+70}{2} = 67.5$  cm  
 c LQ =  $\frac{13}{4}$ th reading =  $55 + \frac{1}{4} \times 5 = 56.25$  cm  
 d UQ =  $9\frac{3}{4}$ th reading =  $75 + \frac{3}{4} \times 35 = 101.25$  cm  
 e IQR =  $101.25 - 56.25$  cm  
 2 a From graph, median = 75 cm  
 b  $77.5 - 72 = 5.5$  cm  
 c 50% of the boxers have a reach with a maximum difference of 5.5 cm



4 a



i From graph, median = 11 cm

ii IQR = 13.7 - 8.1 = 5.6 mins

b  $24 + 36 + p = 92 \Rightarrow p = 32$

$24 + 36 + p + q = 100 \Rightarrow p + q = 40 \Rightarrow q = 8$

5 a From graph, number of students = 1100

b Lower quartile =  $\frac{4200}{4}$ th = 1050th observation  $\approx 39$

Upper quartile = 3150th observation  $\approx 64$

Middle 50% lie between 39 and 64

$\Rightarrow a = 39, b = 64$

c Number getting more than 80  $\approx 4200 - 3900 = 300$

% awarded grade 7  $\approx \frac{300}{4200} \times 100\% = 7.1\%$

6 a 23 mins

b IQR = UQ - LQ = 31 - 16 = 15 mins

c 37 mins

### Exercise 6D

1  $\frac{a + b + 15}{6} = 3 \Rightarrow a + b = 3$

$\frac{(a - 3)^2 + (b - 3)^2 + 1 + 0 + 4 + 4}{6} = \frac{7}{3}$

Solve to find that either  $a = 2, b = 1$  or  $a = 1, b = 2$ .

Given that  $a < b \therefore a = 1, b = 2$ .

2 a Mean =  $\frac{a - 1 + a + a + 2 + a + 3}{4} = \frac{4a + 4}{4} = a + 1$

Variance =  $\frac{(-2)^2 + (-1)^2 + 1^2 + 2^2}{4} + \frac{10}{4} = 2.5$

b Mean =  $a + 1 + 3 = a + 4$

Variance = 2.5

3 a Mean = 9.4

Standard deviation = 1.41

b IQR = 10 - 9 = 1

4  $\frac{2 + 3 + 6 + 9 + x + y}{6} = 6$

$\Rightarrow x + y = 36 - 20 \Rightarrow x + y = 16$  (1)

$\frac{(2 - 6)^2 + (3 - 6)^2 + (6 - 6)^2 + (9 - 6)^2 + (x - 6)^2 + (y - 6)^2}{6} = 10$

$\Rightarrow 16 + 9 + 9 + (x - 6)^2 + (y - 6)^2 = 60$

$\Rightarrow (x - 6)^2 + (y - 6)^2 = 60 - 34 = 26$  (2)

From (1),  $y = 16 - x$ , so

$(x - 6)^2 + (10 - x)^2 = 26$

$\therefore x^2 - 12x + 36 + 100 - 20x + x^2 = 26$

$\Rightarrow 2x^2 - 32x + 110 = 0$

$\Rightarrow x^2 - 16x + 55 = 0$

$\Rightarrow (x + 5)(x - 11) = 0$

$x$  is positive, so  $x = 11$  and  $\therefore y = 5$  (from (1))

The last 2 score sums are 5 and 11

$\therefore$  score sums are 2, 3, 5, 6, 9, 11

Range = 11 - 2 = 9

IQR = 9.5 - 2.75 = 6.75

5 a Mean =  $\frac{4k - 2 + k + k + 1 + 2k + 4 + 3k}{5} = \frac{11k + 3}{5}$

b Variance

$= \frac{\sum x^2}{5} - \left(\frac{\sum x}{5}\right)^2$

$= \frac{(4k - 2)^2 + k^2 + (k + 1)^2 + (2k + 4)^2 + 9k^2}{5} - \left(\frac{11k + 3}{5}\right)^2$

$= \frac{34k^2 - 56k^2 + 96}{25}$

c Mean =  $\frac{11k + 3}{5} - 2 = \frac{11k - 7}{5}$

d The variance will be unchanged as the spread of the data about the mean is unaffected.

6 a Mean =  $\frac{\sum_{r=1}^n (2r - 1)a}{n} = \frac{a}{n} \left(2 \sum_{r=1}^n r - n\right) = \frac{a}{n} [n(n + 1) - n] = \frac{a}{n} \times n^2 = an$

### Exercise 6E

1 a  $P(2, 4, 6, 8) = \frac{4}{2}$

b  $P(3, 6) = \frac{1}{4}$

c  $P(4, 8) = \frac{1}{4}$

d  $P(1, 2, 3, 5, 6, 7) = \frac{3}{4}$

e  $P(1, 2, 3) = \frac{3}{8}$

2  $\frac{30}{150} = \frac{1}{5}$

3 a  $P(A) = \frac{1}{2}$

b  $P(B) = \frac{4}{6} = \frac{2}{3}$

c  $P(A \cup B) = \frac{10}{12} = \frac{5}{6}$

d  $P(A \cap B) = \frac{4}{12} = \frac{1}{3}$

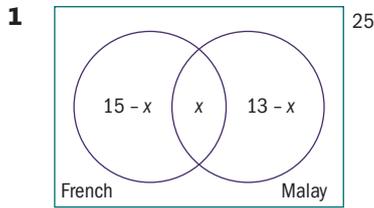
e  $P(A' \cup B) = \frac{10}{12} = \frac{5}{6}$

5 a  $P(A) = \frac{27}{36} = \frac{3}{4}$

b  $P(B) = \frac{18}{36} = \frac{1}{2}$

- c  $P(A \cup B) = \frac{27}{36} = \frac{3}{4}$   
 d  $P(A \cap B) = \frac{18}{36} = \frac{1}{2}$   
 e  $P(A' \cup B') = \frac{18}{36} = \frac{1}{2}$

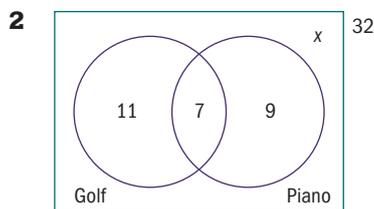
### Exercise 6F



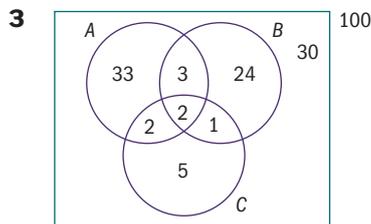
$$15 - x + x + 13 - x + 5 = 25$$

$$\Rightarrow 28 - x = 20 \Rightarrow x = 8$$

$$\therefore P(\text{French and Malay}) = \frac{8}{25}$$



- a  $P(\text{golf but not piano}) = \frac{11}{32}$   
 b  $P(\text{piano but not golf}) = \frac{9}{32}$



- a  $\frac{33}{100} = 0.33$   
 b  $\frac{24}{100} = 0.24$   
 c  $\frac{30}{100} = 0.3$
- 4 a  $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$   
 $= \frac{1}{4} + \frac{1}{8} - \frac{1}{8}$   
 $= \frac{1}{4}$
- b  $P(X \cup Y)' = 1 - P(X \cup Y) = \frac{3}{4}$
- 5 a  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$   
 $= 0.2 + 0.5 - 0.1 = 0.6$
- b  $P(A \cup B)' = 1 - 0.6 = 0.4$
- c  $P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$   
 $= 0.8 + 0.5 - [P(B) - P(A \cap B)]$   
 $= 0.5 + 0.5 - 0.5 + 0.1 = 0.6$

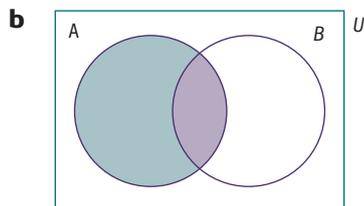
### Exercise 6G

- 1  $P(\text{Sophie and Jerome selected}) = \frac{\binom{8}{2}}{\binom{10}{4}}$   
 $= \frac{2}{15}$
- 2 a  $P(\text{two lines}) = \frac{\binom{3}{2}}{\binom{8}{2}} = \frac{3}{8 \times 7} = \frac{3}{56}$
- b  $P(\text{two different pieces}) = \frac{2 \times \binom{3}{1} \times \binom{5}{1}}{\binom{8}{2}} = \frac{2 \times 3 \times 5}{8 \times 7} = \frac{15}{28}$
- 3 a  $P(R, R, R) = \frac{7}{12} \times \frac{6}{11} \times \frac{5}{10} = \frac{7}{44}$
- b  $P(\text{not all same color})$   
 $= 1 - P(R, R, R) - P(Y, Y, Y)$   
 $= 1 - \frac{7}{44} - \left( \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10} \right)$   
 $= \frac{35}{44}$
- 4 a  $P(\text{all orange}) = \frac{\binom{3}{3}}{\binom{15}{3}} = \frac{10}{455} = \frac{2}{91}$
- b  $P(\text{all different colors})$   
 $= \frac{\binom{4}{1} \times \binom{5}{1} \times \binom{6}{1}}{\binom{15}{3}} = \frac{120}{455} = \frac{24}{91}$
- c  $P(\text{at least one green}) = 1 - P(\text{no green})$   
 $= \frac{\binom{11}{2}}{\binom{15}{3}} = \frac{55}{455} = \frac{11}{91}$
- 5 a i  $P(\text{Bob scores on 1st shoot})$   
 $= P(\text{Bill misses}) \times P(\text{Bob hits})$   
 $= 0.7 \times 0.25 = 0.175$
- ii  $P(\text{Bill scores on 3rd shoot}) = P(\text{Bill misses, Bob misses, Bill misses, Bob misses, Bill hits})$   
 $= 0.7 \times 0.75 \times 0.7 \times 0.75 \times 0.3 \approx 0.0827$
- iii  $P(\text{Bill scores on } n\text{th shoot}) = P(\text{Bill misses, } n\text{ times, Bob misses, } n-1\text{ times, Bob hits})$   
 $= (0.7)^n \times (0.75)^{n-1} \times 0.25$
- b  $P(\text{Bill wins}) = 0.3 + 0.7 \times 0.75 \times 0.3 + 0.7 \times 0.75 \times 0.7 \times 0.75 \times 0.3$   
 $= 0.3 \left( 1 + 0.525 + (0.525)^2 + \dots + 3 \right)$   
 $= 0.3 \times \frac{1}{1 - 0.525}$   
 $\therefore p = \frac{0.3}{1 - 0.525} \Rightarrow p - 0.525p = 0.3$   
 $\Rightarrow p = 0.3 + 0.525p$

c  $P(\text{Bob wins}) = 1 - P(\text{Bill wins})$   
 $= 1 - p$   
 $= 1 - \frac{0.3}{1-0.525} \approx 0.368$

### Exercise 6H

1 a  $P(A \cap B) = P(A) + P(B) - P(A \cup B)$   
 $= 0.4 + 0.6 - 0.7$   
 $= 0.3$



$P(A \cap B') = P(A) - P(A \cap B)$   
 $= 0.4 - 0.3 = 0.1$

c  $P(A' \cup B') = 1 - P(A \cap B) = 0.7$

2  $P(160 < h < 180) = P(h < 180) - P(h < 160)$   
 $= 0.75 - 0.2 = 0.55$

3 a  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$   
 $= 0.6 + 0.55 - 0.2 = 0.95$

b  $P(A' \cap B) = P(B) - P(A \cap B)$   
 $= 0.55 - 0.2 = 0.35$

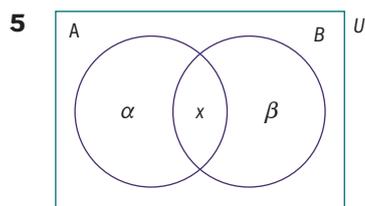
c  $P[(A \cup B) \setminus (A \cap B)] = P(A' \cap B)$   
 $+ P(B' \cap A)$

$= 0.35 + P(A) - P(A \cap B)$   
 $= 0.35 + 0.6 - 0.2 = 0.75$

4 a  $P(A) = 0.5 + 0.2 = 0.7$

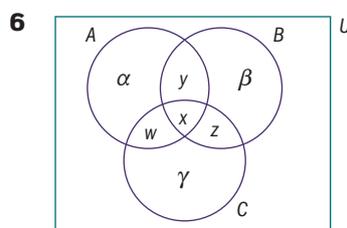
b  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$   
 $\therefore 0.85 = 0.7 + P(B) - 0.2$   
 $\therefore P(B) = 0.35$

c  $P(A' \cap B) = P(B) - P(A \cap B)$   
 $= 0.15$

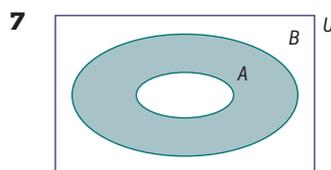


$P(A) \times P(B) = (\alpha + x)(\beta + x)$   
 $= \alpha\beta + x(\alpha + \beta + x)$

But  $\alpha\beta > 0$ , so  $P(A) \times P(B) \geq x(\alpha + \beta + x)$   
 $= P(A \cap B)P(A \cup B)$



$P(A \cup B \cup C) = \alpha + \beta + \gamma + x + y + z + w$   
 $= (\alpha + x + y + w) + (\beta + x + y + z)$   
 $+ (\gamma + w + x + z) - 2x - y - z - w$   
 $= P(A) + P(B) + P(C)$   
 $- [(x + y) + (w + x) + (x + z) - x]$   
 $= P(A) + P(B) + P(C) - P(A \cap B)$   
 $- P(A \cap C) - P(B \cap C) + x$   
 $= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$   
 $- P(B \cap C) + P(A \cap B \cap C)$



$P(B \setminus A) = P(B \cap A') = P(B) + P(A') - P(B \cap A')$   
 $= P(B) + P(A') - 1 = P(B) + 1 - P(A) - 1$   
 $= P(B) - P(A)$

### Exercise 6I

1 a i 0.21

ii  $0.19 + 0.14 = 0.33$

b  $1200 \times 0.21 = 252$

2 a  $\frac{27}{100} = 0.27$

b No. If it was fair we would expect around 16 or 17 occurrences of each number. The spinner appears to be biased towards 1.

c  $0.15 \times 3000 = 450$

3 a  $P(5, 10) = \frac{34 + 68}{100} = \frac{102}{500} = \frac{51}{250}$

b  $P(2, 4, 6, 8, 10, 12) = \frac{6 + 21 + 65 + 63 + 68 + 42}{500} = \frac{265}{500} = \frac{53}{100}$

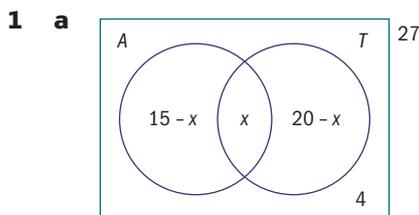
c  $P(2, 4, 6, 8, 10, 12, 5) = \frac{265 + 34}{500} = \frac{299}{500}$

4 a  $P(2, 3, 5, 7) = \frac{4}{10} = \frac{2}{5}$

b  $P(2, 3, 5, 7, 4, 8) = \frac{6}{10} = \frac{3}{5}$

c  $P(3, 6, 9, 4, 8) = \frac{5}{10} = \frac{1}{2}$

### Exercise 6J

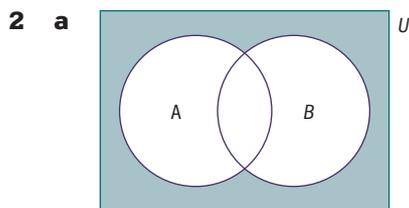


$$15 - x + x + 20 - x = 23 \quad \therefore 35 - x = 23 \quad \therefore x = 12$$

$$P(T \setminus A) = \frac{20 - x}{27} = \frac{8}{27}$$

b  $P(A \cup T) = \frac{3 + 12 + 8}{27} = \frac{23}{27}$

c  $P(T|A) = \frac{P(T \cap A)}{P(A)} = \frac{12}{15} = \frac{4}{5}$



$$P(A' \cap B') = 1 - P(A \cup B)$$

$$\Rightarrow P(A \cup B) = 1 - 0.35 = 0.65$$

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= 0.25 + 0.6 - 0.65 \\ &= 0.2 \end{aligned}$$

b  $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.6} = \frac{1}{3}$

c  $P(B'|A') = \frac{P(B' \cap A')}{P(A')} = \frac{0.35}{1 - 0.25} = \frac{0.35}{0.75} = \frac{7}{15}$

3  $P(\text{roller} | \text{skateboard}) = \frac{P(\text{roller} \cap \text{skateboard})}{P(\text{skateboard})}$   
 $= \frac{0.39}{0.48} = \frac{13}{16}$

4 a  $P(\text{even} | \text{not mult. of 4}) = \frac{P(\text{even} \cap \text{not mult. of 4})}{P(\text{not mult. of 4})}$   
 $= \frac{P(2, 22)}{\frac{6}{8}} = \frac{\frac{2}{8}}{\frac{6}{8}} = \frac{1}{3}$

b  $P(<15|>5) = \frac{P(5 < x < 15)}{P(x < 15)} = \frac{\frac{2}{8}}{\frac{5}{8}} = \frac{2}{5}$

c  $P(<5|<15) = \frac{P(x < 5 \text{ and } x < 15)}{P(x < 15)} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{5}$

d  $P(10 < x < 20 | 5 < x < 25)$   
 $= \frac{P(10 < x < 20 \text{ and } 5 < x < 25)}{P(5 < x < 25)}$   
 $= \frac{\frac{2}{8}}{\frac{4}{8}} = \frac{1}{2}$

5  $P(\text{laptop} | \text{desktop}) = \frac{P(\text{laptop and desktop})}{P(\text{desktop})} = \frac{0.61}{0.95} = \frac{61}{95}$

6  $P(\text{Spanish} | \text{Tech}) = \frac{P(\text{Spanish and Tech})}{P(\text{Tech})} = \frac{0.1}{0.6} = \frac{1}{6}$

7 a  $P(U \text{ and } V) = P(U \cap V) = 0$   
 (mutually exclusive)

b  $P(U|V) = \frac{P(U \cap V)}{P(V)} = 0$

c  $P(U \text{ or } V) = P(U) + P(V) = 0.63$

8  $P(\text{passed 2} | \text{passed 1}) = \frac{P(1 \text{ and } 2)}{P(1)} = \frac{0.35}{0.52} = 0.673$   
 $\therefore 67.3\%$  of those who passed the first also passed the second.

9  $P(\text{white on 2nd} | \text{black on 1st})$

$$= \frac{P(\text{1st black and 2nd white})}{P(\text{black on 1st})}$$

$$= \frac{0.34}{0.47} = \frac{34}{47}$$

10 a  $P(\text{male} \cap \text{left handed}) = \frac{5}{50} = \frac{1}{10}$

b  $P(\text{right handed}) = \frac{43}{50}$

c  $P(\text{right handed} | \text{female})$

$$= \frac{P(\text{right handed and female})}{P(\text{female})} = \frac{\frac{11}{50}}{\frac{13}{50}} = \frac{11}{13}$$

11  $P(\text{other is male} | \text{one is male}) = \frac{P(\text{both male})}{P(\text{one is male})}$

$$= \frac{\frac{1}{4}}{P(\text{not 2 females})}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

### Exercise 6K

1  $P(A \cap B) = 0.24 = 0.4 \times 0.6 = P(A) \times P(B)$

$\therefore A$  and  $B$  independent

$$P(B \cap C) = 0.15 \neq P(B) \times P(C)$$

$\therefore B$  and  $C$  not independent

2  $P(A \cap B) = P(\text{Red Queen}) = \frac{2}{52} = \frac{1}{26}$

$$P(A) \times P(B) = \frac{4}{52} \times \frac{1}{2} = \frac{1}{26}$$

$\therefore A$  and  $B$  independent

$$P(B \cap C) = P(\text{red face card}) = \frac{6}{52} = \frac{3}{26}$$

$$P(B) \times P(C) = \frac{1}{2} \times \frac{12}{52} = \frac{6}{52} = \frac{3}{26}$$

$\therefore B$  and  $C$  are independent

$$P(A \cap C) = P(\text{Queen and face card})$$

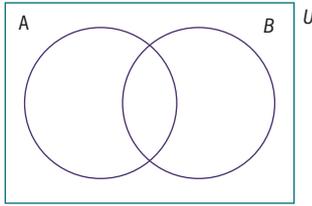
$$= P(\text{Queen}) = \frac{4}{52} = \frac{1}{13}$$

$$P(A) \times P(C) = \frac{4}{52} = \frac{1}{13}$$

$$= \frac{1}{13} \times \frac{3}{13} \neq \frac{1}{13}$$

$\therefore A$  and  $C$  are not independent

3



$$\begin{aligned} \text{a } P(A \cap B') &= P(A) - P(A \cap B) \\ &= P(A) - P(A)P(B) \\ &= P(A)(1 - P(B)) \\ &= P(A)P(B') \end{aligned}$$

$\therefore A$  and  $B'$  are not independent

$$\begin{aligned} \text{b } P(A' \cap B) &= P(B) - P(A \cap B) \\ &= P(B) - P(A)P(B) \\ &= P(B)(1 - P(A)) \\ &= P(A')P(B) \end{aligned}$$

$\therefore A'$  and  $B$  are independent

$$\begin{aligned} \text{c } P(A' \cap B') &= 1 - P(A \cup B) \\ &= 1 - [P(A) + P(B) - P(A \cap B)] \\ &= 1 - P(A) - P(B) + P(A \cap B) \\ &= (1 - P(A))(1 - P(B)) \\ &= P(A')P(B') \end{aligned}$$

$\therefore A'$  and  $B'$  are independent

$$\text{4 } P(B|A) = \frac{P(B \cap A)}{P(A)} \Rightarrow P(B \cap A) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{5}{6} = \frac{1}{3} + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{5}{6} = \frac{1}{3} + P(B) - \frac{1}{4} \Rightarrow P(B) = \frac{5}{6} - \frac{1}{3} + \frac{1}{4} = \frac{3}{4}$$

$$P(A) \times P(B) = \frac{1}{3} \times \frac{3}{4} = \frac{1}{4} = P(A \cap B)$$

$\therefore A$  and  $B$  independent

$$\begin{aligned} \text{5 a } P(A) \times P(B) &= P(A \cap B) \\ \Rightarrow P(B) &= \frac{P(A \cap B)}{P(A)} = \frac{0.18}{0.45} = \frac{2}{5} = 0.4 \end{aligned}$$

$$\begin{aligned} \text{b } P(A \cap B') &= P(A) - P(A \cap B) \\ &= 0.45 - 0.18 = 0.27 \end{aligned}$$

$$\begin{aligned} \text{c } P(A' \cap B') &= P(A') \times P(B') \\ &= 0.55 \times 0.6 = 0.33 \end{aligned}$$

$$\begin{aligned} \text{6 a } P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= 3a - \frac{5}{8} = P(A)P(B) \\ \Rightarrow 2a^2 - 3a + \frac{5}{8} &= 0 \Rightarrow 16a^2 - 24a + 5 = 0 \end{aligned}$$

$$\Rightarrow (4a - 5)(4a - 1) = 0 \Rightarrow a = \frac{5}{4} \text{ or } a = \frac{1}{4}$$

Since  $P(A)$  and  $P(B)$  must be  $\leq 1$ ,  $P(A) = \frac{1}{4}$ ,

$$P(B) = \frac{1}{2}$$

$$\text{7 a } P(T, 6) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

$$\text{b } P(H, \text{even}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\text{c } P(H, 3 \text{ or } 6) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

$$\text{8 a } P(x, x, x, \text{even}) = 1 \times 1 \times 1 \times \frac{1}{2} = \frac{1}{2}$$

$$\text{b } P(x, x, x, 0 \text{ or } 5) = 1 \times 1 \times 1 \times \frac{2}{10} = \frac{1}{5}$$

$$\begin{aligned} \text{c } P(\text{divisible by } 4) &= P(\text{last 2 digits divisible by } 4) \\ &= P(x, x, 0, 0) + P(x, x, 0, 4) \\ &\quad + P(x, x, 0, 8) + \dots \\ &\quad + P(x, x, 9, 6) \\ &= \frac{25}{100} = \frac{1}{4} \end{aligned}$$

9 Since there is a large number of integers, can assume  $P(\text{odd}) = P(\text{even}) = \frac{1}{2}$

Suppose we select  $n$ . Then  $P(n \text{ even}) = \left(\frac{1}{2}\right)^n$

$$\therefore P(\text{at least one odd}) = 1 - \left(\frac{1}{2}\right)^n$$

$$\text{Need } 1 - \left(\frac{1}{2}\right)^n > 0.92$$

$$\Rightarrow \left(\frac{1}{2}\right)^n < 0.08$$

$$n \log 0.5 > \log 0.08$$

$$\Rightarrow n > 3.64 \therefore \text{need to select 4 integers}$$

10  $P(\text{Julia fails to score a winner}) = 0.45$

$$\therefore P(\text{Julia fails } n \text{ times in a row}) = (0.45)^n$$

$$\therefore P(\text{at least one winner in } n \text{ shots}) = 1 - (0.45)^n$$

$$1 - (0.45)^n > 0.999$$

$$\Rightarrow (0.45)^n < 0.001$$

$$\Rightarrow n > \frac{\log 0.001}{\log 0.45} \Rightarrow n > 8.65$$

$\therefore$  Julia needs to hit 9 shots

### Exercise 6L

$$\text{1 } P(\text{Rain, not late}) = 0.2 \times 0.6 = 0.12$$

$$\text{2 } P(\text{Correct diagnosis}) = 0.85 \times 0.98 + 0.15 \times 0.12 = 0.851$$

$$\text{3 } P(1 \text{ Score out of } 2) = 0.75 \times 0.15 + 0.25 \times 0.8 = 0.3125$$

$$\begin{aligned} \text{4 a } P(B') &= \frac{1}{3} \times \frac{2}{5} + \frac{2}{3} \times \frac{1}{2} \\ &= \frac{2}{15} + \frac{1}{3} \\ &= \frac{7}{15} \end{aligned}$$

$$\begin{aligned} \text{b } P(A' \cup B') &= \frac{2}{3} \times 1 + \frac{1}{3} \times \frac{2}{5} \\ &= \frac{2}{3} + \frac{2}{15} \\ &= \frac{12}{15} + \frac{2}{15} \end{aligned}$$

$$\begin{aligned} \text{5 a } P(\text{orange, orange, orange}) &= \frac{18}{30} \times \frac{17}{29} \times \frac{16}{28} \\ &= \frac{3}{5} \times \frac{17}{29} \times \frac{4}{7} \\ &= \frac{204}{1015} \end{aligned}$$

$$\begin{aligned} \text{b } P(\text{at least one purple}) &= 1 - P(\text{all orange}) \\ &= 1 - \frac{204}{1015} = \frac{811}{1015} \end{aligned}$$

$$\begin{aligned} \text{c } P(\text{more orange than purple}) &= P(\text{o, o, o}) + P(\text{o, p, o}) \\ &\quad + P(\text{o, o, p}) + P(\text{p, o, o}) \\ &= \frac{204}{1015} + \frac{18}{30} \times \frac{12}{29} \times \frac{17}{28} + \frac{18}{30} \times \frac{17}{29} \times \frac{12}{28} \\ &\quad + \frac{12}{30} \times \frac{18}{29} \times \frac{17}{28} \\ &= \frac{204}{1015} + \frac{459}{1015} = \frac{663}{1015} \end{aligned}$$

$$\text{6 a } P(\text{R, R, R}) = \frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} = \frac{2}{17}$$

$$\text{b } P(\text{H, H, H}) = \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} = \frac{11}{850}$$

$$\text{c } P(\text{all same suit}) = \frac{52}{52} \times \frac{12}{51} \times \frac{11}{50} = \frac{22}{425}$$

$$\text{d } P(\text{faces in same suit}) = \frac{12}{52} \times \frac{2}{51} \times \frac{1}{50} = \frac{1}{5525}$$

### Exercise 6M

$$\begin{aligned} \text{1 a } P(\text{even}) &= \frac{1}{2} \times \frac{4}{9} + \frac{1}{2} \times \frac{3}{5} \\ &= \frac{2}{9} + \frac{3}{10} = \frac{47}{90} \end{aligned}$$

$$\begin{aligned} \text{b } P(\text{first box} \mid \text{even}) &= \frac{P(\text{first box} \cap \text{even})}{P(\text{even})} \\ &= \frac{\frac{1}{2} \times \frac{4}{9}}{\frac{47}{90}} = \frac{20}{47} \end{aligned}$$

$$\text{2 a } P(\text{defective}) = 0.6 \times 0.05 + 0.4 \times 0.02 = 0.038$$

$$\begin{aligned} \text{b } P(\text{first machine} \mid \text{defective}) &= \frac{P(f \cap d)}{P(d)} \\ &= \frac{0.6 \times 0.05}{0.038} = \frac{0.03}{0.038} \\ &= 0.789 \end{aligned}$$

$$\text{3 a } P(V) = 0.6 \times 0.35 + 0.4 \times 0.75 = 0.51$$

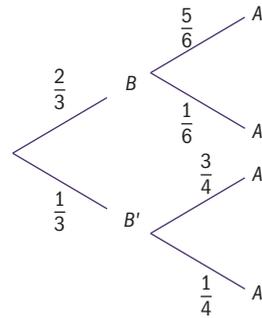
$$\text{b } P(V' \mid G) = 0.25$$

$$\begin{aligned} \text{4 a } P(\text{BB from 2nd box}) &= \frac{16}{30} \times \frac{13}{20} \times \frac{12}{19} + \frac{14}{30} \times \frac{12}{20} \times \frac{11}{19} \\ &= \frac{4344}{30 \times 20 \times 19} = 0.381 \end{aligned}$$

$$\begin{aligned} \text{b } P(\text{1st box W} \mid \text{both from 2nd box W}) &= \frac{P(\text{WWW})}{P(\text{WW 2nd box})} \end{aligned}$$

$$\begin{aligned} &= \frac{\frac{14}{30} \times \frac{8}{20} \times \frac{7}{19}}{\frac{16}{30} \times \frac{7}{20} \times \frac{6}{19} + \frac{14}{30} \times \frac{8}{20} \times \frac{7}{19}} = \frac{784}{1456} = \frac{49}{91} = \frac{7}{13} \end{aligned}$$

5 a



$$\begin{aligned} \text{b i } P(A) &= \frac{2}{3} \times \frac{5}{6} + \frac{1}{3} \times \frac{3}{4} = \frac{5}{9} + \frac{1}{4} \\ &= \frac{20+9}{36} = \frac{29}{36} \end{aligned}$$

$$\text{ii } P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{2}{3} \times \frac{5}{6}}{\frac{29}{36}} = \frac{20}{29}$$

$$\text{iii } P(B' \mid A') = \frac{P(B' \cap A')}{P(A')} = \frac{\frac{1}{3} \times \frac{1}{4}}{\frac{2}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{4}} = \frac{3}{7}$$

$$\text{6 a } P(6) = \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{6} = \frac{1}{3} + \frac{1}{12} = \frac{5}{12}$$

$$\begin{aligned} \text{b } P(\text{unbiased} \mid \text{not 6}) &= \frac{P(\text{unbiased and not 6})}{P(\text{not 6})} \\ &= \frac{\frac{1}{2} \times \frac{5}{6}}{\frac{1}{2} \times \frac{5}{6} + \frac{1}{2} \times \frac{1}{3}} = \frac{5}{7} \end{aligned}$$

$$\text{7 a } P(\text{non-smoker}) = 0.18 \times 0.1 + 0.82 \times 0.8 = 0.674$$

$$\begin{aligned} \text{b } P(\text{lung problems} \mid \text{heavy smoker}) &= \frac{P(\text{lung problems and heavy smoker})}{P(\text{heavy smoker})} \\ &= \frac{0.18 \times 0.7}{0.18 \times 0.7 + 0.82 \times 0.05} = 0.754 \end{aligned}$$

$$\begin{aligned} \text{8 a } P(R) &= \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{3}{8} + \frac{1}{3} \times \frac{3}{5} \\ &= \frac{2}{9} + \frac{1}{8} + \frac{1}{5} = \frac{197}{360} \end{aligned}$$

$$\text{b } P(C \mid R) = \frac{P(C \cap R)}{P(R)} = \frac{\frac{1}{3} \times \frac{3}{5}}{\frac{197}{360}} = \frac{1}{5} \times \frac{360}{197} = \frac{72}{197}$$

$$\begin{aligned} \text{9 a } P(\text{on time}) &= 0.45 \times 0.95 + 0.2 \times 0.90 \\ &\quad + 0.35 \times 0.80 \\ &= 0.8875 \end{aligned}$$

$$\begin{aligned} \text{b } P(A \mid \text{on time}) &= \frac{P(A \text{ and on time})}{P(\text{on time})} \\ &= \frac{0.45 \times 0.95}{0.8875} \approx 0.482 \end{aligned}$$

$$\text{c } P(B \mid \text{late}) = \frac{P(B \text{ and late})}{P(\text{late})} = \frac{0.2 \times 0.1}{1 - 0.8875} \approx 0.178$$

$$\begin{aligned}
 10 \text{ a } P(\text{Jar 2}, B) &= \frac{5}{15} \times \frac{4}{14} \times \frac{5}{11} + \frac{5}{15} \times \frac{10}{14} \times \frac{6}{11} \\
 &\quad + \frac{10}{15} \times \frac{5}{14} \times \frac{6}{11} + \frac{10}{15} \times \frac{9}{14} \times \frac{7}{11} \\
 &= \frac{1330}{15 \times 14 \times 11} = \frac{19}{33}
 \end{aligned}$$

$$\begin{aligned}
 \text{b } P(\text{Jar 1} = \text{PP} \mid \text{Jar 2} = \text{P}) &= \frac{P(\text{PPP})}{P(\text{Jar 2} = \text{P})} \\
 &= \frac{\frac{5}{15} \times \frac{4}{14} \times \frac{6}{11}}{1 - P(B)} \\
 &= \frac{5 \times 4 \times 6}{980} = \frac{6}{49}
 \end{aligned}$$

$$\begin{aligned}
 \text{c } P(\text{Jar 1} = \text{BB} \mid \text{Jar 2} = \text{P}) \\
 &= \frac{P(\text{BBP})}{P(\text{Jar 2} = \text{P})} = \frac{\frac{10}{15} \times \frac{9}{14} \times \frac{4}{11}}{\frac{980}{15 \times 14 \times 11}} \\
 &= \frac{18}{49}
 \end{aligned}$$

$$11 \text{ a } P(\text{male}) = 0.1 \times 0.6 + 0.65 \times 0.7 + 0.25 \times 0.3 = 0.59$$

$$\begin{aligned}
 \text{b } P(\text{management} \mid \text{male}) &= \frac{P(\text{male and management})}{P(\text{male})} \\
 &= \frac{0.1 \times 0.6}{0.59} = 0.102
 \end{aligned}$$

$$\begin{aligned}
 \text{c } P(\text{marketing} \mid \text{female}) &= \frac{P(\text{female and marketing})}{P(\text{female})} \\
 &= \frac{0.25 \times 0.7}{1 - 0.59} \approx 0.427
 \end{aligned}$$

$$\begin{aligned}
 12 \text{ P(second machine} \mid \text{D')} \\
 &= \frac{P(\text{second machine and D'})}{P(\text{D'})} \\
 &= \frac{0.35 \times 0.97}{0.5 \times 0.96 + 0.35 \times 0.97 + 0.15 \times 0.94} \\
 &\approx 0.353
 \end{aligned}$$

$$\begin{aligned}
 13 \text{ P}(320, 320 \mid S = 160) \\
 &= \frac{P(320, 320, 160)}{P(S = 160)} \\
 &= \frac{\frac{8}{20} \times \frac{7}{19} \times \frac{12}{18}}{\frac{12}{20} \times \frac{11}{19} \times \frac{10}{18} + \frac{12}{20} \times \frac{8}{19} \times \frac{11}{18} + \frac{8}{20} \times \frac{12}{19} \times \frac{11}{18} + \frac{8}{20} \times \frac{7}{19} \times \frac{12}{18}} \\
 &= \frac{672}{4104} = \frac{28}{171}
 \end{aligned}$$

$$14 \text{ P}(S) = \frac{4}{11} \times 0.9 + \frac{4}{11} \times 0.6 + \frac{3}{11} \times 0.2 = 0.6$$

$$\begin{aligned}
 15 \text{ P(vowel)} &= \frac{3}{7} \times \frac{2}{6} \times \frac{1}{5} + \frac{3}{7} \times \frac{4}{6} \times \frac{2}{5} \\
 &\quad + \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} + \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} \\
 &= \frac{90}{7 \times 6 \times 5} = \frac{3}{7}
 \end{aligned}$$



### Review exercise

- 1 Mode = 6, so smallest set is 6, 6,  $x$ ,  $y$   
 Median = 7, so set is 6, 6, 8,  $y$   
 Mean = 8, so  $\frac{6+6+8+y}{4} = 8 \Rightarrow 20 + y = 32$   
 $\therefore$  set is 6, 6, 8, 12

$$\begin{aligned}
 2 \text{ } A \text{ and } B \text{ independent} \\
 \Rightarrow P(A \mid B) = P(B) \Rightarrow P(B) = \frac{1}{3}
 \end{aligned}$$

Let  $P(A) = x$ :

$$\begin{aligned}
 P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
 \therefore \frac{11}{12} &= x + \frac{1}{3} - x \times \frac{1}{3} \\
 \Rightarrow \frac{2}{3}x + \frac{1}{3} &= \frac{11}{12} \\
 \frac{2}{3}x = \frac{7}{12} &\Rightarrow x = \frac{7}{8}
 \end{aligned}$$

3 From graph:

- a Median = 68 kg
- b Middle 50% = 61 – 77 kg
- c There are 36 students

$$4 \text{ a } \binom{12}{3} = 220$$

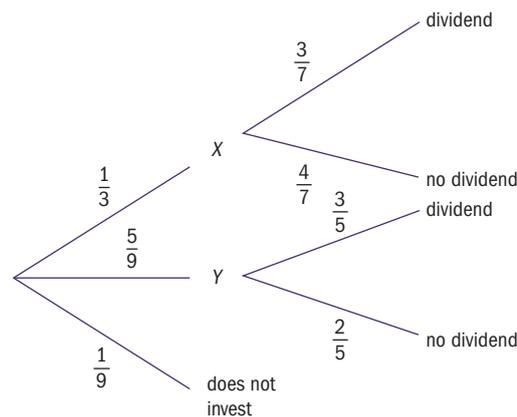
b Probability =  $1 - P(\text{both on committee})$

$$\begin{aligned}
 &= 1 - \frac{\binom{10}{1}}{\binom{12}{3}} = 1 - \frac{10}{220} = \frac{11}{22}
 \end{aligned}$$

c  $P(2 \text{ girls, } 1 \text{ boy}) + P(3 \text{ girls})$

$$\begin{aligned}
 &= \frac{\binom{5}{2} \times \binom{7}{1}}{220} + \frac{\binom{5}{3}}{220} \\
 &= \frac{80}{220} = \frac{4}{11}
 \end{aligned}$$

5 a



$$\begin{aligned}
 \text{b } P(\text{dividend}) &= \frac{1}{3} \times \frac{3}{7} + \frac{5}{7} \times \frac{3}{5} \\
 &= \frac{1}{7} + \frac{1}{3} = \frac{10}{21}
 \end{aligned}$$

$$\begin{aligned}
 \text{c } P(Y \mid \text{dividend}) &= \frac{P(Y \text{ and dividend})}{P(\text{dividend})} = \frac{\frac{5}{9} \times \frac{3}{5}}{\frac{10}{21}} \\
 &= \frac{7}{10}
 \end{aligned}$$

6  $P(\text{1st G} \mid \text{2nd G}) = \frac{P(\text{GG})}{P(\text{2nd G})}$

$$= \frac{\frac{5}{12} \times \frac{4}{11}}{\frac{3}{12} \times \frac{5}{11} + \frac{4}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{4}{11}}$$

$$= \frac{20}{15 + 20 + 20} = \frac{20}{55} = \frac{4}{11}$$

7 a  $P(\text{prime}) = \frac{6}{36} = \frac{1}{6}$

b  $P(\text{even}) = \frac{27}{36} = \frac{3}{4}$

c  $P(\text{multiple of 3}) = \frac{20}{36} = \frac{5}{9}$

d  $P(\text{divisible by 6} \mid \text{even}) = \frac{P(\text{divisible by 6 and even})}{P(\text{even})}$

$$= \frac{\frac{15}{36}}{\frac{3}{4}} = \frac{5}{9}$$

8 a i mean =  $\frac{\sum mi}{n} = \frac{540}{30} = 18$

ii Variance =  $\frac{\sum mi^2}{n} - \left(\frac{\sum mi}{n}\right)^2$

$$= \frac{9990}{30} - 18^2 = 333 - 324 = 9$$

$\therefore \text{SD} = 3$

b No, as 95% of the students should get marks within 2 standard deviations of the mean, i.e. between 12 and 24, and 99.7% within 3 standard deviations of the mean, i.e. between 9 and 27.

9 a  $\frac{3n+1}{2}$

b There are  $n$  number of the form  $3k$  as  $k$  runs from 1 to  $n$ . But every other one is even, so number of odd numbers is  $\frac{n+1}{2}$

Hence  $P(\text{divisible by 3}) = \frac{\frac{n+1}{2}}{\frac{3n+1}{2}} = \frac{n+1}{3n+1}$



Review exercise

1 Mean height =  $\frac{23 \times 168 + 17 \times 171 + 8 \times 163 + 20 \times 177}{68}$

$$= 170.8\text{cm}$$

2 a  $\frac{10!}{3!3!2!} = 50400$

b  $P(\text{S} \text{-----}) = \frac{9!}{50400} = \frac{15120}{50400} = \frac{3}{10}$

c  $P(\text{-----consonant})$

$$= 1 - \frac{3!3!2!}{50400} - \frac{3!3!}{50400}$$

$$= 1 - \frac{5040}{50400} - \frac{10080}{50400}$$

$$= 1 - \frac{15120}{50400} = \frac{7}{10}$$

3 a  $\frac{10 \times 10 \times 5}{10 \times 10 \times 10} = \frac{1}{2}$

b  $P(\text{abc divisible by 7}) = \frac{142}{1000} = \frac{71}{500}$

c  $P(\text{abc} = \text{perfect square}) = P(x^2, 1 \leq x \leq 31)$

$$= \frac{31}{1000}$$

4 Probability =  $1 - 0.93 \times 0.93$

$$= 0.1351$$

5 a Mean = 337.5 cm

b Standard deviation = 132.64 km

6

7

8 a Mean = 4.69

Standard deviation = 0.552

b

RBC	CF
3.4	0
3.8	7
4.2	22
4.6	58
5.0	80
5.4	107
5.8	120

Median = 60th observation = 4.64

c Number of children with RBC > 5.5  $\approx 120 - 110 = 10$

9 a  $P(\text{all stats books in first 6 places}) = \frac{14!}{\frac{7!4!3!}{20!}}$

$$= \frac{14!6!}{20!} = \frac{1}{38760}$$

b  $P(\text{all calculus books together}) = \frac{14!}{\frac{6!7!4!3!}{20!}}$

$$= \frac{14!7!}{20!} = \frac{7}{38760}$$

10 a  $P(\text{Keith wins bet i.e. } A \text{ winning})$

$$= \frac{4}{11} \times 0.4 + \frac{3}{11} \times 0.55 + \frac{4}{11} \times 0.75$$

$$\approx 0.568$$

b  $P(A \text{ played a higher rank} \mid A \text{ lost}) = \frac{\frac{4}{11} \times 0.6}{1 - 0.568}$

$$= 0.505$$